only talks about a faculty to assess numerical quantity, the target article conveys a rather simplistic view of an "innate" and fully hardwired system that extracts numerical information just like a reflex. However, such a narrow definition does not hold for any cognitive capability and is not maintained by protagonists of the "number sense" (Anobile et al. 2016c; Burr & Ross 2008; Viswanathan & Nieder 2013). Physiological faculties are plastic (subject to maturation and/or learning processes); they are embedded in – and interact with – other faculties. The finding that the number faculty interacts with general magnitude representations can therefore not refute its existence.

Second, a key argument of the article is that varying the number of items in a set inevitably changes physical stimulus parameters. Although this is undisputed, it is far too premature to conclude that investigations of numerical representations are therefore a priori useless. The two main reasons are as follows:

1. Potential sensitivity to simple sensory parameters is not specific to number investigations but pervasive to all investigations targeting abstract representations. Semantic groups can only be tested with specific stimulus representatives. Continuous magnitudes are, of course, no different in that respect. Resorting to continuous magnitude therefore does not solve the problem.

2. In contrast to the impression caused by omissions in the target article, many researchers painstakingly selected their stimuli and went to great lengths to demonstrate number representations. Because it is not physically possible to equate all possible stimulus parameters at the same time, the best way is to control – unbeknown to the subject – one parameter after the other in separate stimulus configurations. If the subject responds equally to systematically varied numerosity stimuli, it is safe to conclude that the subject responds to number (Nieder 2016). One of the main research agendas over the last two decades therefore was to test numerosity representations over a broad range of stimuli and formats. For example, humans have recently been shown to be far more sensitive to numerosity than to continuous magnitudes in dot displays (Cicchini et al. 2016). Greater sensitivity to changes in numerosity was present both spontaneously and in tasks where participants were explicitly instructed to judge continuous parameters of the dot displays. Therefore, humans extract number information based on dedicated mechanisms. In addition, studies using controlled stimuli with conditioned animals demonstrated clear numerosity judgments. For example, the seminal monkey study by Brannon and Terrace (1998) controlled for item location, overall surface area, item size, and item type. Later, Nieder et al. (2002) controlled for item position, overall item area, overall item circumference, high and low density, item type, shape-like item configurations, and linear item arrangements, the latter one also abolishing convex hull. Monkeys also extracted the number of elements that appeared sequentially one-by-one and matched it to the number in spatial dot arrays (Nieder et al. 2006). In this sequential presentation format, temporal parameters such as duration, rhythm, and accumulated intensity have been controlled for and were neglected by the monkeys. Moreover, monkeys assessed number also independently of the sensory modality and discriminated both the number of sequential visual dots and auditory sounds within the same session (Jordan et al. 2008a; Nieder 2012). The animals did not care about non-numerical magnitude changes and responded to number information. Similar results have been obtained in preschool children (Barth et al. 2005). In sum, evidence for the capability of nonverbal subjects to represent numerical quantity is stronger than ever.

Third, another unfortunate omission of Leibovich et al. concerns abstract number representations in the brain. The single-neuron code underlying number representations has been addressed over the past years with a broad range of controlled stimuli. These studies in animals showed surprisingly abstract number representations ("number neurons"). As reviewed in Nieder (2016), number neurons recorded in monkeys performing the aforementioned numerical tasks were tuned to preferred numerosities while being largely insensitive to changing sensory features. Number neuron responses in prefrontal cortex (PFC), and to some extent in the intraparietal sulcus (IPS), generalized across spatial features in visual item arrays (Nieder et al. 2002), spatio-temporal visual presentation formats (Nieder et al. 2006), and also visuo-auditory presentation formats to signal numerosity supramodally (Nieder 2012). Moreover, in monkeys trained to associate shapes with numerosities, neurons signaled the numerical meaning of signs (Diester & Nieder 2007). Number neurons were present even if monkeys were not trained on number (Viswanathan & Nieder 2013). After training, PFC showed improved responses to numerosity during active discrimination, whereas ventral intraparietal area (VIP) neurons remained stable (Viswanathan & Nieder 2015). Of course, such highly generalized responses of number neurons cannot (and should not) be expected to be the only code for numerical quantity. Abstract number information can also be extracted from population activity (Ramirez-Cardenas et al. 2016; Tudusciuc & Nieder 2007). Collectively, these single-neuron recordings strongly support the idea of a dedicated number faculty residing in a parieto-frontal network, with striking similarities between numerical representations in nonhuman and human primates (Nieder 2016).

Leibovich et al. also err when claiming that only one study (Castelli et al. 2006) had directly compared brain areas during number and continuous magnitude comparison tasks. For example, Pinel et al. (2004) found that number and size, but not luminance, activated overlapping parietal regions during functional imaging. More directly, single-cell recordings in monkeys that discriminated numerical, spatial, and sensory magnitudes in one session showed that coding was largely dissociated at the single-neuron level (Eiselt & Nieder 2016; Tudusciuc & Nieder 2009). Therefore, numerical representations are based on distributed coding by single neurons that are anatomically intermingled within the same cortical area.

Contrary to the claim of the target article, overwhelming evidence supports a dedicated number faculty that operates independent from continuous magnitude. The target article's attempt to reduce number judgments to simple magnitude representations is a lost case. Far from being put to rest, the number faculty is alive and kicking.

The contributions of non-numeric dimensions to number encoding, representations, and decision-making factors

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Abstract: Leibovich et al. suggest that congruency effects in number perception (biases towards smaller, denser, etc., dots) are evidence for the number's dependence on these dimensions. I argue that they fail to differentiate between effects at three distinct levels of number perception – encoding, representations, and decision making – and that differentiating between these allows the number to be independent from, but correlated with, non-numeric dimensions.

Visual and auditory number stimuli inherently correlate with dimensions such as size, density, rate, and so forth, and observers are sometimes biased towards these dimensions: Changing the density of a collection of dots also changes which set observers believe to be more numerous. Leibovich et al., following the footsteps of recent findings reporting such "congruency effects," argue that number may be entirely or partly dependent on these dimensions – that there is no innate number sense independent from our perception of density, convex hull, size, and so forth.

However, their critique leaves a key question open: At what level of processing do non-numeric dimensions exert their hold on number? There are at least three independent possibilities, and only one of them is consistent with the central claim against an independent number sense.

The first possibility is that number is encoded using low-level visual features, such as orientation, contrast, spatial frequency, and so forth, which are shared with other dimensions, rather than out of its own dedicated feature detectors (e.g., Dakin et al. 2011 vs. Burr & Ross 2008). For example, consider that face perception strongly depends on a unique mix of low- and high-spatial frequency, and, therefore, changing frequency information also changes which emotion is most strongly perceived (Vuilleumier et al. 2003). In this same manner, there are now many reasons to suspect that number encoding depends on features such as low-spatial frequency (Dakin et al. 2011), and that it may even depend on distinct features at different levels of crowding (Anobile et al. 2014). Thus, manipulating density (i.e., low-spatial frequency information) can result in changes in number perception, not because of number being represented *as* density, but rather because of their shared dependence on identical low-level features. Congruency effects, therefore, could be interpreted as positive results describing the nature of low-level features used to encode number, not as evidence against its dependence on non-numeric dimensions. At the very least, claims to number's non-independence must first account for the shared low-level features.

The second possibility is that number and non-numeric dimensions compete for the same decision-making component, such as putting a common load on working memory or yielding similar response conflicts (Hurewitz et al. 2006; Odic et al. 2016; Van Opstal & Verguts 2013). Once again, consider an analogy: congruency effects found in the Stroop effect do not imply that color perception is dependent on and statistically learned from reading ability, but rather that multiple dimensions can compete for the same response. Because we know that density and area perception tend to be more accurate in adults compared with number, there is plenty of reason to think that these dimensions will win a "horse race" for the same response as number, creating congruency effects without any shared representations (Hurewitz et al. 2006). Consistent with this, my colleagues and I have demonstrated that number and time perception only correlate when individual differences in working memory are not controlled for (Odic et al. 2016). More recently, we have found that the effect of non-numeric dimensions such as contour length is entirely eliminated when Stroop-like response conflicts are alleviated (Picon & Odic, in preparation). Together, these results suggest that many demonstrated congruency effects could be response conflicts, and that any claim for dependence between number perception and non-numeric dimensions should first control for these factors.

Finally, the third possibility for the link between number and non-numeric dimensions – and one that is most consistent with the claims of Leibovich et al. – is that number may be (antecedently) represented on the same representational scale as other dimensions, either by being directly represented as, for example, area, or alternatively by being represented on a domain-general, unitless magnitude scale that simply codes for *more* versus *less* (Cantrell & Smith 2013; Lourenco & Longo 2010; Walsh 2003). Although Leibovich et al. suggest that statistical learning eventually separates number from these dimensions, their theory requires that – from birth until some later age – numerical information is represented in one of these two ways. But, as reviewed previously, evidence for shared representations must first control for the possibility of shared encoding or decision-making factors; given that the majority of existing work fails to do so, what is the evidence for shared/unified representations? Perhaps the most convincing case cited by Lebovich et al. is that of Tudusciuc and Nieder (2007), who found neurons in the parietal cortex that respond to both number and length. But a closer inspection of their data reveals that these neurons often code in opposing ways: The same neuron may code for small numbers, but very long lengths, or vice versa, running contrary to the idea of a shared scale and instead consistent with a set of overlapping population coding neurons that play different roles for each dimension.

Another approach at demonstrating shared representations is to simultaneously measure number and the candidate shared dimensions, such as area, length, density, and time; if number shares the scale for these representations, any individual and developmental variability within number should be accounted for by differences in these other dimensions. Recently, my lab followed this logic through and tested 2- to 12-year-old children and adults on these five discrimination tasks. We found that number develops independently from area, length, density, and time, which in turn develop independently from it stretching back to age 2 (Odic 2017; see also Odic et al. 2013). Hence, unless the kind of proposed statistical learning proposed leads to complete differentiation by age 2, it is difficult to imagine how these results could be obtained without a significant independence between number and area, length, density, and time perception.

To conclude, Leibovich et al. make a bold claim – that congruency effects are illustrative of number's dependence on non-numeric dimensions – but their critique fails to account for the possibility that these effects stem from shared encoding or decision-making components, not shared representations. Future work exploring number's dependence should carefully disentangle the contributions of other dimensions to encoding and decision making, as these levels are not constitutive of the independent representations of number.

Numerical magnitude evaluation as a foundation for decision making

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Abstract: The evaluation of magnitudes serves as a foundation not only for numerical and mathematical cognition, but also for decision making. Recent theoretical developments and empirical studies have linked numerical magnitude evaluation to a wide variety of core phenomena in decision making and challenge the idea that preferences are driven by an innate, universal, and stable sense of number or value.

Leibovich et al.'s critique of the "number sense" theory is timely and has implications beyond the literature on numerical and mathematical cognition. Numerical magnitude perception also plays a critical role in decision making, as it shapes how people trade off outcomes that vary in size, probability, and timing. Moreover, recent theoretical developments and empirical findings from the study of decision making have shown that evaluations of numerical magnitudes are neither innate, nor universal, nor stable, but vary substantially across countries, individuals, and contexts.

The evaluation of numerical magnitudes in decision making. The evaluation of numerical magnitudes is critical to decision making and implicitly forms the core of many important theories of choice (e.g., Kahneman & Tversky 1979). For example, an individual faced with multiple job offers needs to compare (among other